Discrete Mathematics Recitation Course 5

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張玟翔

5-1

The Basics of Counting

5-1 Ex.20

- How many positive integers less than 1000
  - f) are divisible by neither 7 nor 11?
  - g) have distinct digits?
  - h) have distinct digits and are even?

- $999 - (142 + 90 - 12) = 779$
- $9 + (9 \cdot 9) + (9 \cdot 9 \cdot 8) = 9 + 81 + 648 = 738$
- $738 - (5 + 5 \cdot 8 + 5 \cdot 8 \cdot 8) = 373$

5-1 Ex.30

- How many strings of eight English letters are there
  - f) that start with the letters BO (in that order), if letters can be repeated? 26^6
  - g) that start and end with the letters BO (in that order), if letters can be repeated? 26^4
  - h) that start or end with the letters BO (in that order), if letters can be repeated? $26^6 + 26^6 - 26^4$

5-1 Ex.39

- A **palindrome** is a bit string whose reversal is identical to the string. How many bit strings of length n are palindromes?

- If n is even, $2^{n/2}$
- If n is odd, $2^{(n+1)/2}$

5-1 Ex.44

- How many bit strings of length 10 contain either five consecutive 0s or five consecutive 1s?

- $2^5 + 5 \cdot 16 = 112$
- $112 + 112 - 2 = 222$

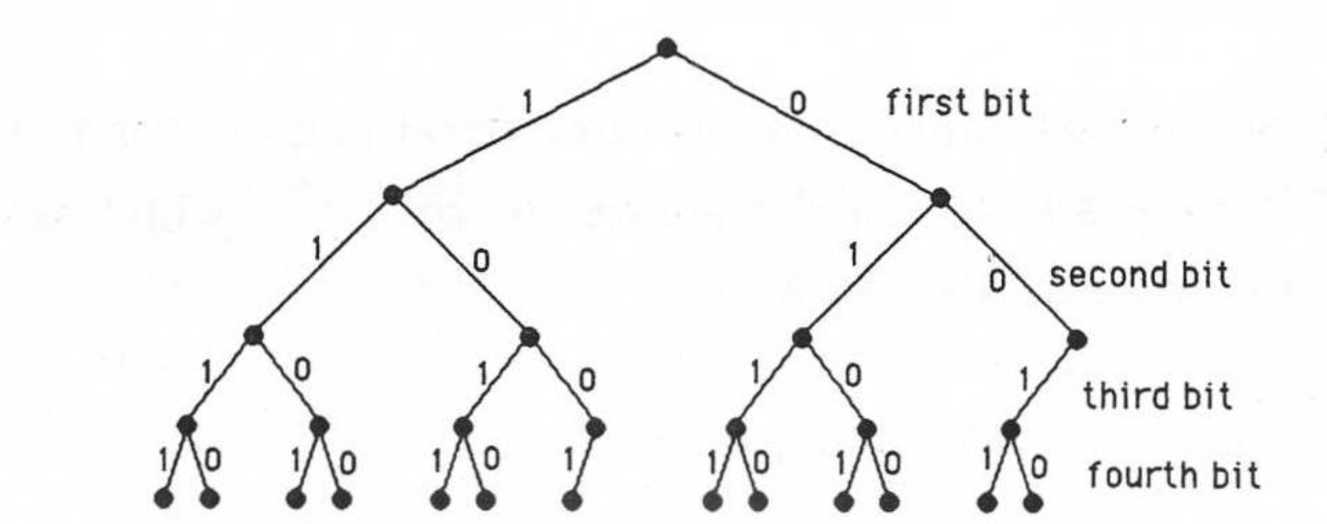
5-1 Ex.48

- How many different initials can someone have if a person has at least two, but no more than five, initials? Assume that each initial is one of the 26 letters of the English language.

- $26^2 + 26^3 + 26^4 + 26^5$

5-1 Ex.52

- Use a tree diagram to find the number of bit strings of length four with no three consecutive 0s.



5-2

The Pigeonhole Principle

5-2 Ex.16

- How many numbers must be selected from the set {1, 3, 5, 7, 9, 11, 13, 15} to guarantee that at least one pair of these numbers add up to 16?

- 4 groups: {1, 15}, {3, 13}, {5, 11}, {7, 9}
- 5 integers

5-2 Ex.29

- Show that there are at least six people in California (population:36million) with the same three initials who where born on the same day of the year (but not necessarily in the same year). Assume that everyone has the initials.

- $26^3 * 366 = 6{,}432{,}816$
- $\lceil 36{,}000{,}000/6{,}432{,}816 \rceil = 6$

5-2 Ex.32

- A computer network consists of six computers. Each computers is directly connected to at least one of the other computers. Show that there are at least two computers in the network that are directly connected to the same number of other computers.

- Let $K(x)$ be the number of computers that computer x is connected to. The possible values of $K(x)$ is 1, 2, 3, 4, 5.
- 5 integers, 6 computers

5-2 Ex.36

- Prove that at a party where there are at least two people, there are two people who knows the same number of other people there. (We assume that "knowing" is symmetric)

- Let $K(x)$ be the number of other people at the party that x knows. Then $K(x) = \{0,1,2,\ldots,n-1\}$, where $n \geq 2$.
- n pigeons(x) and n pigeon holes($K(x)$).
- However, it is impossible for both 0 and $n-1$ to be in the range of K.
- n pigeons and $n-1$ pigeon holes.

5-2 Ex.42

- Let $n_1, n_2, \ldots, n_t$ be positive integers. Show that if $n_1 + n_2 + \cdots + n_t - t + 1$ objects are placed into t boxes, then for some $i, i = 1, 2, \ldots, t$, the ith box contains at least n_i objects.

- Suppose this statement is not true. Then for each i, the ith box contains at most $n_i - 1$ objects.
- Adding, we have at most $(n_1 - 1) + (n_2 - 1) + \cdots + (n_t - 1) = n_1 + n_2 + \cdots + n_t - t$ objects in all.
- Contradiction!